20MAT-2A

GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS) SRIKAKULAM

REACCREDITED WITH NAAC 'A' GRADE

B.Sc (Supplementary) Semester-III

(W.e.f.2022-23 Admitted Batch)

TITLE: Abstract Algebra TIME: 2 Hrs

Marks : 60 Date :13-12-2024

SECTION –A

Answer ALL of the following question: 5X8M=40M

1) a) Show that the set Q^+ of all positive rational numbers forms an abelian group under the Composition defined by 'o' such that $ob = \frac{ab}{3}\ for\ a, b \in Q^+$.

OR

b) In a group G($\neq \emptyset$) $for\ a, b, x, y \in G$ the equation ax=b and ya=b have unique solutions

2)a)The necessary and sufficient condition for a finite complex H of a group G to be a subgroup of G is a,b$\in$ H $\Rightarrow ab^{-1} \in H$

OR

b) State & prove Lagrange's theorem.

3)a) Prove that a subgroup H of a group G is a normal subgroup of G iff each left coset of H in G is a right coset of H in G.

OR

b) State and prove fundamental theorem on homomorphism groups.

4)a) State and prove Cayley's theorem.

OR

b) Prove that every subgroup of cyclic group is cyclic.

5)a)Prove that every finite integral domain is a field

OR

b) Prove that every subgroup of cyclic group is cyclic.

SECTION-B

Answer any FIVE of the following questions **5X4M=20M**

6) . In a group G for every a in G,a^2=e, prove that G is abelian.

7) Find the order of each element of the group G =Z_6= {0, 1, 2, 3, 4, 5}, the composition being addition modulo 6.

8) Prove that for a non-empty subset H of a group G to be a subgroup of G then H^{-1} = H.

9) Prove that if G is a group and H is a subgroup of index 2 in G then H is normal subgroup of G.

10) . Prove that if H_1 and H_2 are two normal subgroups of a group G then $H_1 \cap H_2$ is also a normal subgroup of G.

11) Prove that if f is a homomorphism of a group G into a group G', then the Kernel of f is a normal subgroup of G.

12) Examine whether the following Permutations are even or odd i) $\begin{pmatrix} 1 2 3 4 5 6 7 8 9 \\ 6 1 4 3 2 5 7 8 9 \end{pmatrix}$ ii) $\begin{pmatrix} 1 2 3 4 5 6 7 \\ 3 2 4 5 6 7 1 \end{pmatrix}$

13) Show that every cyclic group is abelian.

14) Prove that a ring R has no zero divisors iff the cancellation laws hold in R.

15) Prove that the ideals of a field F are only {0} and F itself