23MAT-MJ5-2A

GOVERNMENT COLLEGE FOR WOMEN(AUTONOMOUS),SRIKAKULAM
REACCREDITED WITH NAAC 'A' GRADE

B.Sc (Regular) Semester-III
(2023-24 Admitted Batch)

Paper : Group theory and problem solving sessions **Time : 3 Hrs**
Marks : 60 **Date : 09-12-2024**

SECTION –A

Answer ALL of the following question: **5X8M=40M**

1) a) If G is a set of even integers G={ .....-4,-3,-2,-1,0,1,2,3,4.....} Then Prove That G is an abelian group with usual addition as the operation

OR

b) In a group G(≠∅) for a,b,x,y∈G the equation ax=b and ya=b have unique solutions

2)a)The necessary and sufficient condition for a finite complex H of a group G to be a subgroup of G is a,b H ⇒ab∈H

OR

b) The union of two subgroups of a group is a subgroup iff one is contained in the other.

3)a) Prove that a subgroup H of a group $G$is a normal subgroup of G iff product of two right (left) cosets of H in G is a right(left) cosets of H in G.

OR

b)If H is a subgroup of G and N is a normal subgroup of G then i) HN is a normal subgroup of H ii) N is a normal subgroup of HN

4)a) The necessary and sufficient condition for a homomorphism f of a group G onto a group G' With kernel K to be an monomorphism of G into G' .Prove that K ={e}

OR

b) State and prove Fundamental Theorem of homomorphism of groups

5)a) let Sn be the permutation group on n symbols then prove that The n! Permutations are in n!/2 are even and n!/2 are odd permutations

OR

5)b) State and prove Cayley's theorem .

SECTION-B

Answer any FIVE of the following questions **5X4M=20M**

6) Show that the set $G= \{x/x= 2^a3^b \ and \ a, b\in Z\}$ is a group under multiplication

7) Define order of an element .In a group$G$,prove that if$a\in G$then $0(a)=o(a^{-1})$.

8) If H and $K$are two subgroups of a group G then prove that HK is a sub group of G iff $HK=KH$

9)If a,b are any two elements of a group (G,.) and H any subgroup of G then Ha=Hb ⇔ a b^{-1} H and aH =bH ⇔ a^{-1}b H

10) If $G$is a group and H is a sub group of index 2 in G then prove that H is a normal sub group of G.

11)If N,M are normal subgroups of G .Then NM is also a normal subgroup of G

12) A Homomorphism f: G→G' is one-one then k={e} ,where k=kernel of f.

13) If for a group G,f: G→G' is given by f(x) =x2 ,xG is homomorphism then prove that G is an abelian

14) Examine whether the following Permutations are even or odd i) 1 2 3 4 5 6 7 8 9 6 1 4 3 2 5 7 8 9 ii)1 2 3 4 5 6 73 2 4 5 6 7 1

15) Express the product (2 5 4) (1 4 3) (2 5) as a product of disjoint cycles and find its inverse

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