

23MAT-MJ8-2A
GOVERNMENT COLLEGE FOR WOMEN (AUTONOMOUS) , SRIKAKULAM
REACCREDITED WITH NAAC 'A' GRADE

B.Sc (Regular) Semester-III
(2023-24 Admitted Batch)

Paper Special Functions and Problem Solving Sessions
Marks : 60

Time : 3 Hrs
Date : 13-12-2024

SECTION –A

Answer ALL of the following question:

5X8M=40M

1(a) Prove That $\Gamma(n) \Gamma(n + \frac{1}{2}) = \frac{\sqrt{\pi}}{2^{2n-1}} \Gamma(2n)$

(OR)

(b) State and Prove Relation between Beta and Gamma function

2(a) show that $x=0$ is an irregular singular point $x=-1$ is a regular singular point of $x^2 (x + 1)^2 y'' + (x^2 - 1) y' + 2y = 0$

(OR)

(b) Find the series solution of $(1 - x^2)y'' + 2y = 0, y(0) = 4, y'(0) = 5$

3(a) State Prove Rodrigues Formula for Hermite polynomial

(OR)

(b) Find the first few terms of Hermite polynomials

4(a) Prove That $(1 - 2xh + h^2)^{-1/2} = \sum_{n=0}^{\infty} h^n P_n(x)$

(OR)

(b) Prove That $(2n+1)xP_n = (n+1)P_{n+1} + nP_{n-1}$

5(a) Prove That $\sqrt{\frac{\pi x}{2}} J_{3/2}(x) = \frac{1}{x} \sin x - \cos x$

(OR)

(b) Prove That $x J_n'(x) = n J_n(x) - x J_{n+1}(x)$

SECTION-B

Answer any FIVE of the following questions

5X4M=20M

6) Evaluate $\int_0^{\infty} \frac{x^2}{1+x^4} dx$

7) Prove That i) $T_n(-1) = (-1)^n$ ii) $T_{2n}(0) = (-1)^n$ iii) $T_{2n+1}(0) = 0$

8) Find the radius of convergence of the following series $\sum \frac{n^n x^n}{n!}$

9) Find the radius of convergence of $\frac{x}{2} + \frac{1.3}{2.5} x^2 + \frac{1.3.5}{2.5.8} x^3 + \dots$

10) Prove That $H_n'(x) = 2nH_{n-1}(x)$

11) Prove That $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$

12) Find the value of $P_0(x), P_1(x), P_2(x), P_3(x)$

13) Prove That i) $P_n'(1) = \frac{1}{2} n(n+1)$ ii) $P_n'(-1) = (-1)^{n-1} \frac{1}{2} n(n+1)$

14) $J_{-n}(x) = (-1)^n J_n(x)$

15) Prove That $\frac{d}{dx} [x^{-n} J_n(x)] = x^{-n} J_{n+1}(x)$